



**Problem 1.** The toric variety  $V = V(xy - zw) \subset \mathbb{C}^4$ .

Consider the affine variety that is defined by the polynomial  $f(x, y, z, w) := xy - zw$ .

- i) Identify a 3-dimensional algebraic torus as subset of  $V$ .
- ii) Which Laurent monomials extend to functions of  $V$ ? Describe the lattice points  $(a, b, c) \in \mathbb{Z}^3$  such that a Laurent monomial  $t_1^a \cdot t_2^b \cdot t_3^c$  extends to a function on  $V$ .
- iii) Describe the cone  $C \subset \mathbb{R}^3$  that contains the exponents of the Laurent monomials that extend to  $V$  in terms of primitive  $\mathbb{Z}^3$ -vectors that generate  $C$ .