TECHNISCHE UNIVERSITÄT MÜNCHEN
Zentrum Mathematik

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Discrete Geometry: Lattice Polytopes (winter 2014/15)

— Problem Set 12 (January 13, 2016) —

Problem 1. The product of a sum of four squares is the sum of four squares.
Let $\xi_1, \xi_2, \xi_3, \xi_4, \eta_1, \eta_2, \eta_3, \eta_4$ be integers. Show that there are integers $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ such that

$$(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2)(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2) = \zeta_1^2 + \zeta_2^2 + \zeta_3^2 + \zeta_4^2.$$

Problem 2. Sum of three squares does not suffice.
Give an example of a positive integer that is not the sum three squares of integers.

Problem 3. Approximation of irrational numbers.
Let $\lambda \in \mathbb{N}$ and $\alpha \in \mathbb{R}$. Then there exist $p, q \in \mathbb{Z}$ with $q > \lambda$ such that

$$\|\alpha - \frac{p}{q}\| \leq \frac{C}{q^2}$$

for some $0 < C < 1$ that is independent of λ and α.