Riemann Surfaces
Exercise Sheet 6

Exercise 1. (meromorphic functions with one simple pole)
Let $M$ be a compact Riemann surface and let $f$ be a meromorphic function on $M$ with only one simple pole. Show that $M$ is conformally equivalent to $\hat{\mathbb{C}}$.

Exercise 2. (symmetries of tori and coverings)
Let $M \subseteq \mathbb{R}^3$ be an embedded torus obtained by rotating a circle in the $xz$-plane with center on the $x$-axis around the $z$-axis.
(a) Topologically, what surface is obtained by identifying pairs of points on $M$ that are symmetric with respect to a $180^\circ$ rotation around the $x$-axis?
(b) Topologically, what surface is obtained by identifying pairs of points on $M$ that are diametrically opposite, i.e., images of each other under the map $x \mapsto -x$.

Exercise 3. (handles and crosscaps)
Show that $\mathbb{R}P^1 \# T^2 = \mathbb{R}P^1 \# \mathbb{R}P^1 \# \mathbb{R}P^1$.

Hint: Come up with a cut-and-glue procedure. Alternatively, make drawings of deforming surfaces. For the second approach, note that $\mathbb{R}P^1$ is a Möbius band with a disk glued to the boundary, and $\mathbb{R}P^1 \# \mathbb{R}P^1$ is the Klein bottle.