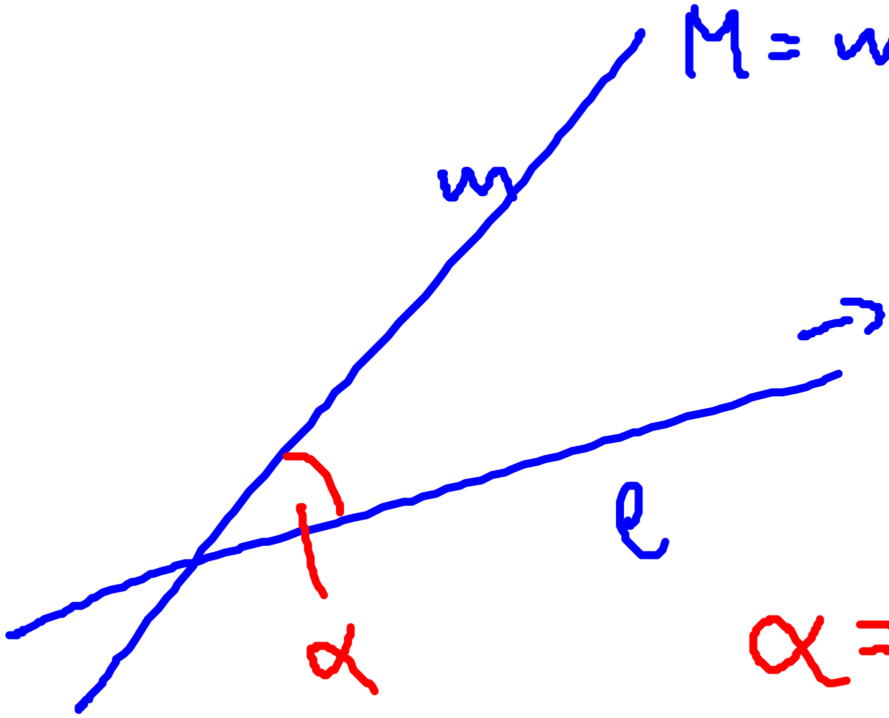
Letztes Mal:

Formel von Laguerre



$M = m \times l_\infty$

$L = l \times l_\infty$

$$\alpha = \frac{1}{2i} \ln (L, M; I, J)$$

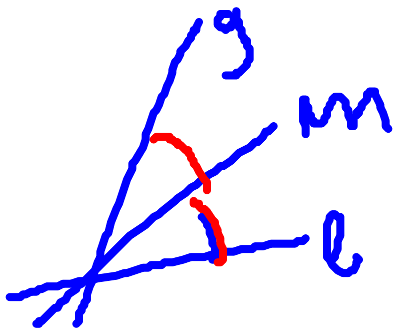
Eigenschaften des Winkels $\alpha_{\ell m} := \frac{1}{2i} \ln(L,M;I,J)$

- Mehrdeutigkeit modulo π, da $\ln$ mehrdeutig modulo $2\pi i$ ist.

- Vertauschung von ℓ, m kehrt den Winkel in sein negatives um.

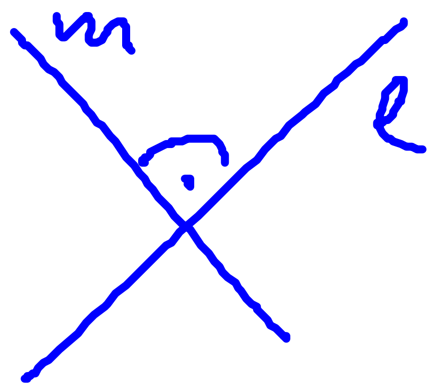
$$\begin{array}{ccc} (L,M;I,J) & = & \dfrac{1}{(M,L;I,J)} \\ \ln\downarrow & & \downarrow\ln \\ \alpha_{\ell m} & = & -\alpha_{\ell m} \end{array}$$

- Winkel sind additiv (modulo π)



$$\alpha_{\ell m} + \alpha_{mg} \overset{\text{mod } \pi}{=} \alpha_{\ell g} \quad \overset{\ln}{\rightsquigarrow} \quad (L,M;I,J)\cdot(M,G;I,J) = (L,G;I,J)$$

- Senkrecht stehen



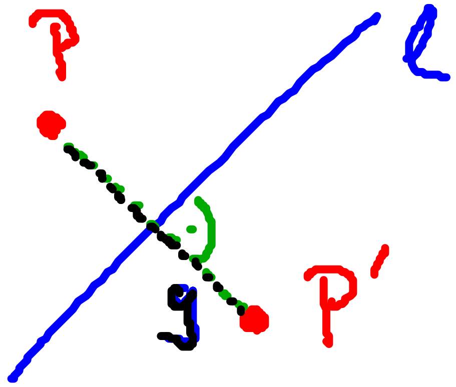
$$\frac{\pi}{2} = \frac{1}{2i} \ln(L, M; l, J)$$

$$\Leftrightarrow \quad i\pi = \ln(L, M; l, J)$$

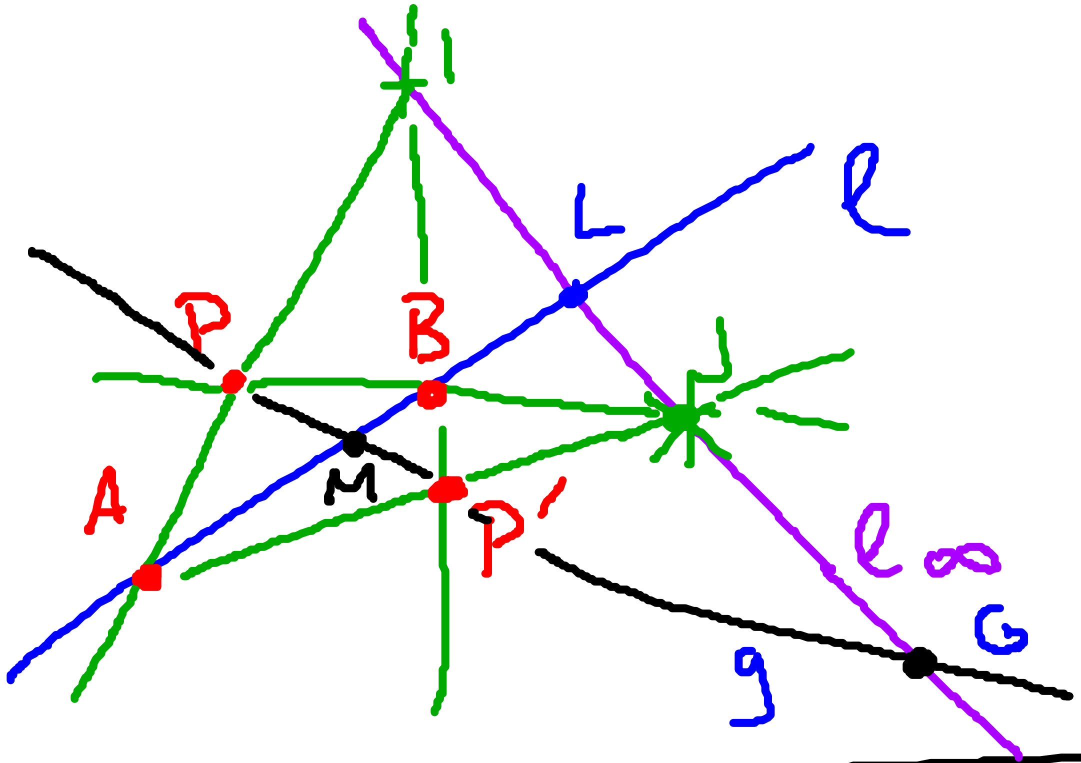
$$\Leftrightarrow \quad e^{i\pi} = (L, M; l, J)$$

$$\Leftrightarrow \quad -1 = \underbrace{(L, M; l, J)}_{\text{harmonische Lage}}$$

Euklidische Konstruktionen:

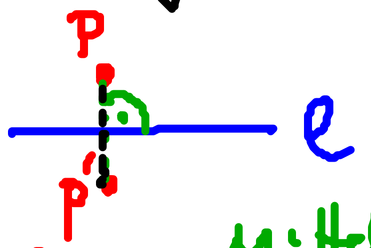


- Konstruktion eines Spiegelbildes



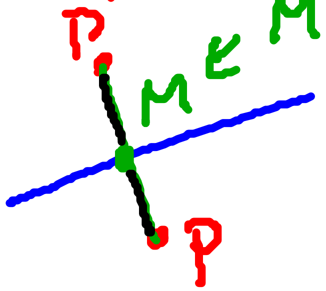
Zwei Eigenschaften von P'

(1)



$(L, G; I, J) = -1$

(2)



$(P, P'; M\, G) = -1$

1: $\ell_A = P \times I$

2: $\ell_B = P \times J$

3: $A = \ell_A \times \ell$

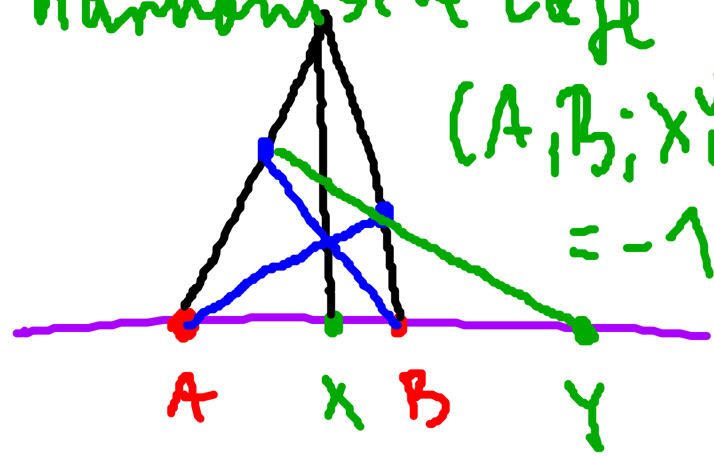
4: $B = \ell_B \times \ell$

5: $g_A = A \times J$

6: $g_B = B \times I$

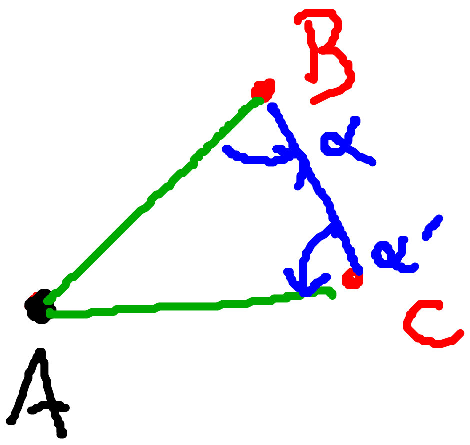
7: $P' = g_A \times g_B$

harmonische Lage



$(A, B; X, Y) = -1$

- gleichen Abstand



$$|AC| = |AB|$$

$$\Leftrightarrow \alpha = \alpha'$$

$$\left.\frac{B-A}{B-C} \Big/ \frac{C-B}{C-A} \in \mathbb{R} \right\} \text{ in } \mathbb{C}P^1 \text{ gedacht}$$

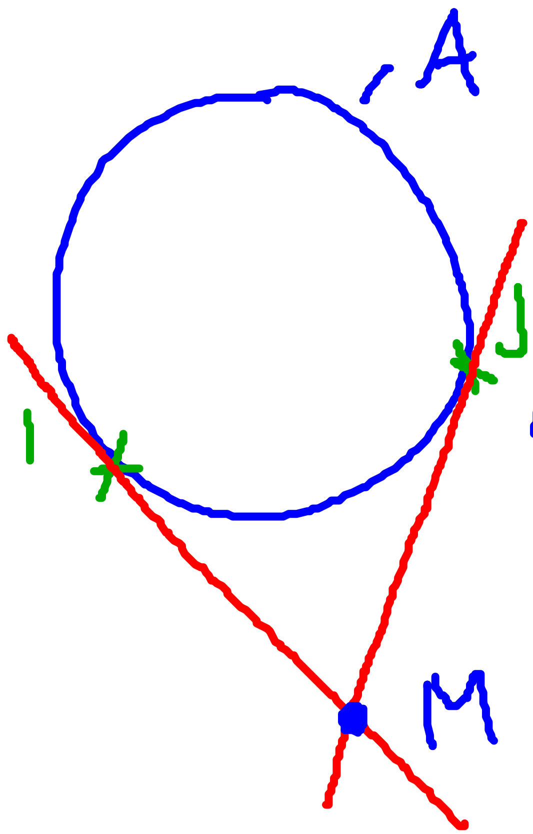
$$\Rightarrow \frac{(B-A)(C-A)}{(B-C)^2} \in \mathbb{R}$$

übliche Übersetzung

$$[BAI][CAI][BCJ]^2 = [BAJ][CAJ][BCI]^2$$

Weitere Konstruktionen:

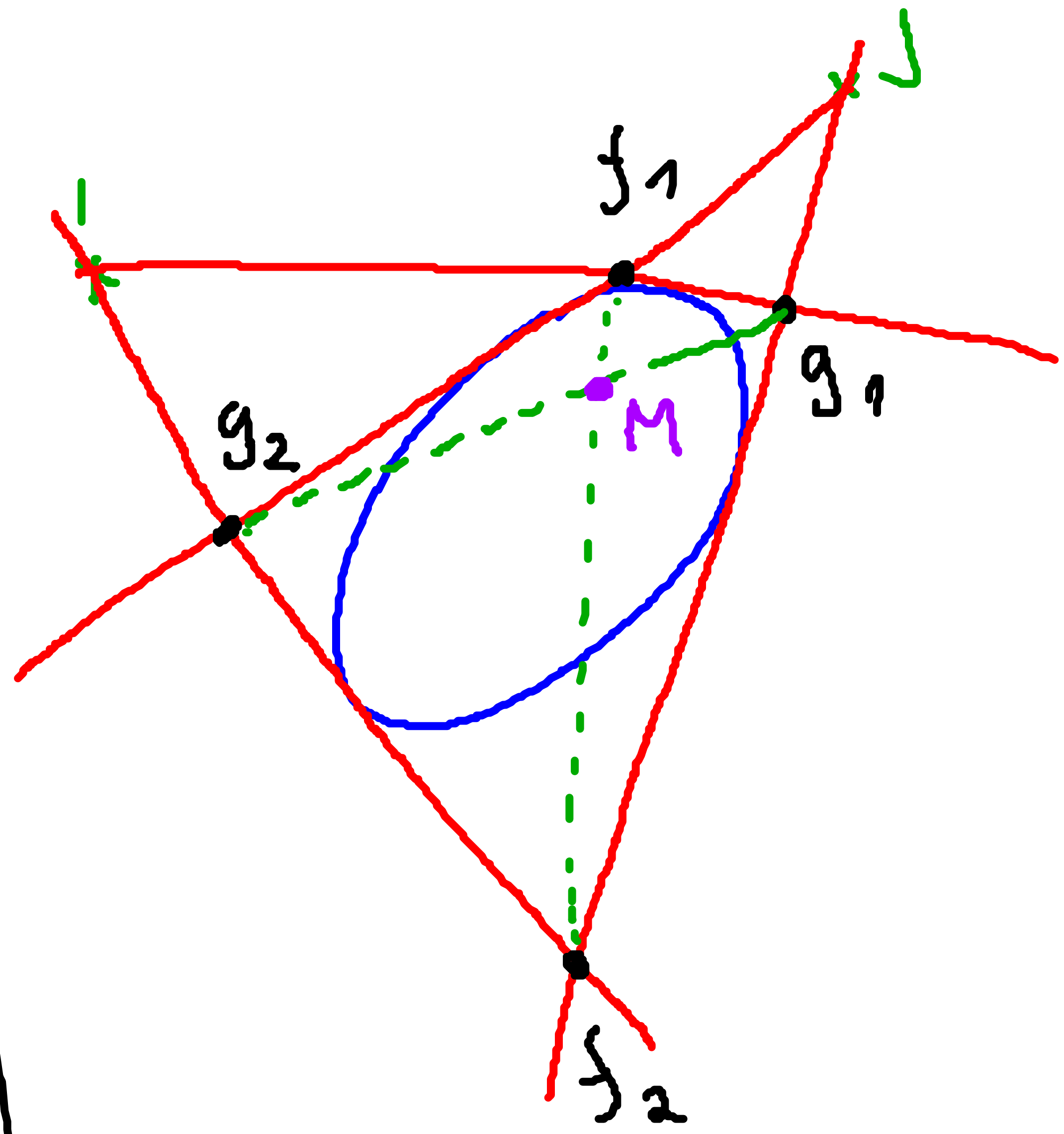
- Kreismittelpunkt



$M = AI \times AJ$

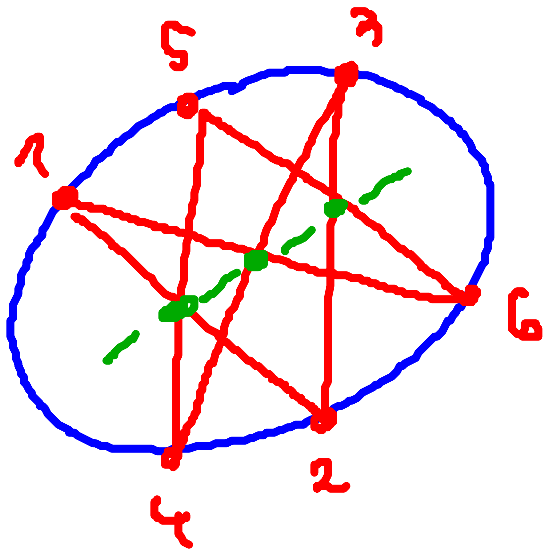
Beweis: Übung

- Brennpunkte eines Kegelschnittes



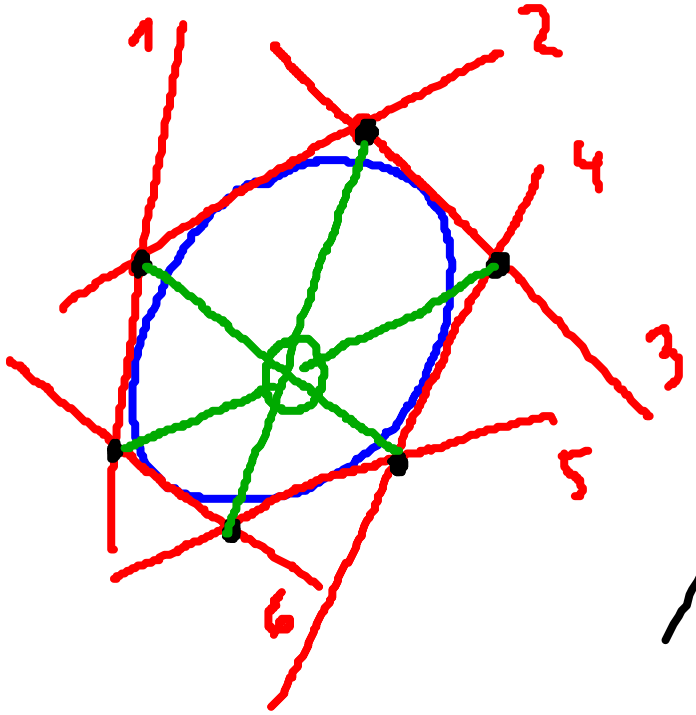
Schritt 1

Satz von Pascal



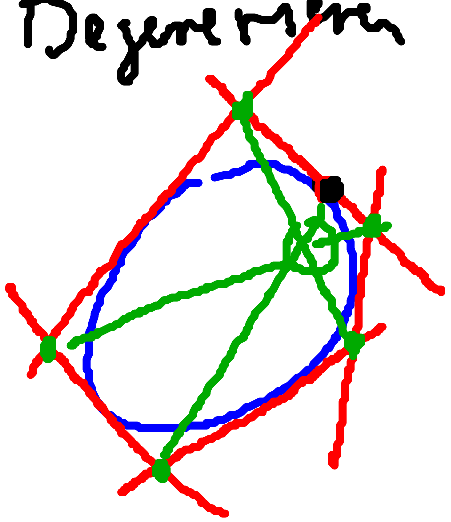
Schritt 2: Dualisieren

Satz von Brianchon

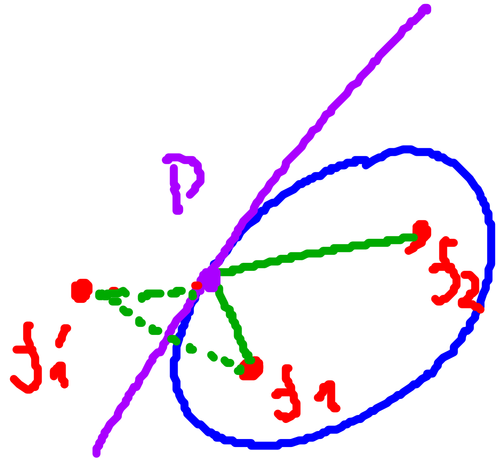


Schritt 3

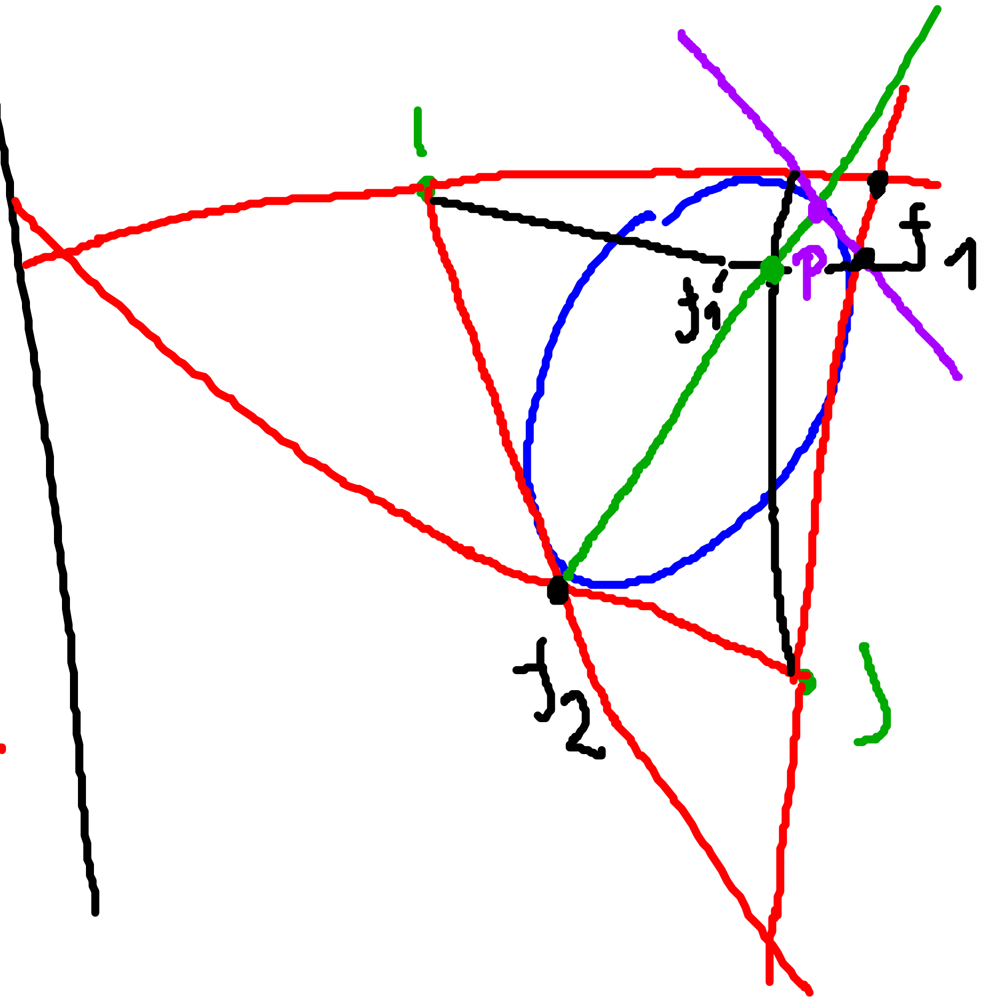
Degenerieren



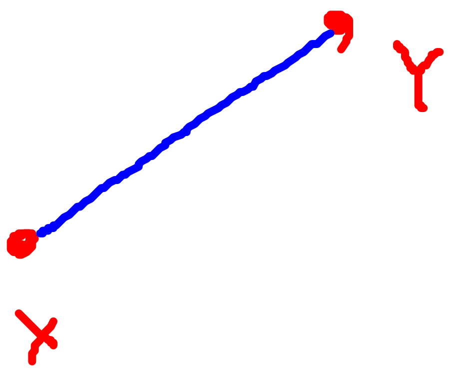
Schritt 4: Charakterisiere Brennpunkte



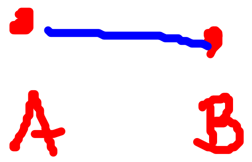
Schritt 5: „Beweis"



Abstand zwischen zwei Punkten.



In $\mathbb{C}$ $\quad \sqrt{(x-y)\overline{(x-y)}} = |X,Y|$



$|AB| = 1$

$$|XY| = \frac{\overbrace{\sqrt{[XYIJ][XYJI]}}^{|XY|}\ \overbrace{[AIJ]}^{-2i}\,\overbrace{[BIJ]}^{-2i}}{\underbrace{\sqrt{[ABIJ][ABJI]}}_{|AB|=1}\ \underbrace{[XIJ]}_{-2i}\,\underbrace{[YIJ]}_{-2i}}$$