Prep. course:

25 29

M T M T F

10 10 X 2

2

4

1 2 5

Mo. Fr

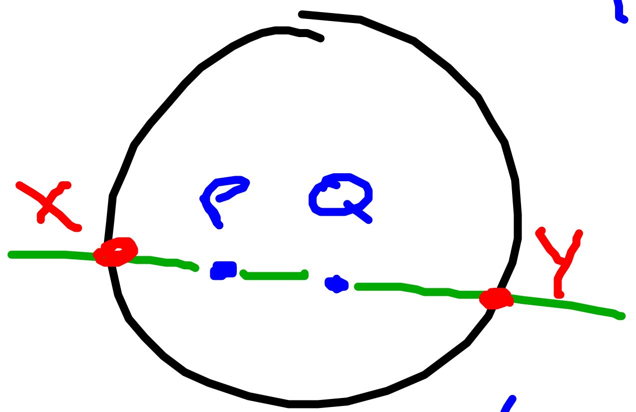
10k

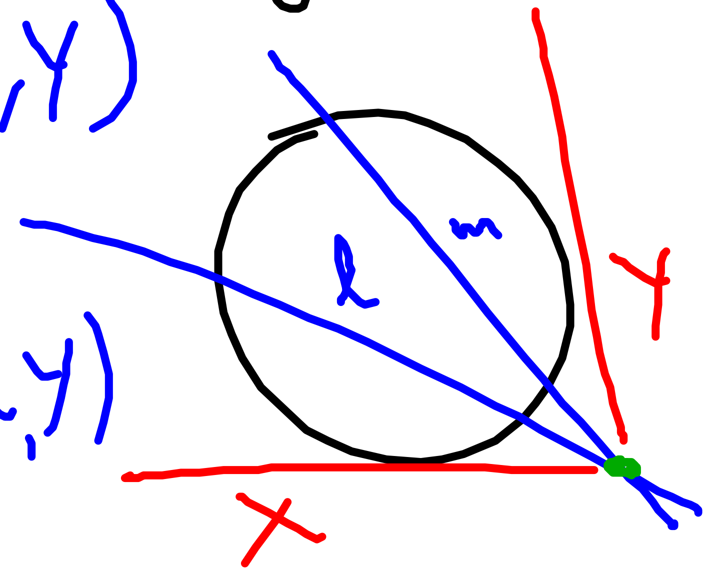
$12^{00} - 16^{00}$

Prep course at Friday $12^{00} - 16^{00}$

Last Time: Cayley-Klein geom.

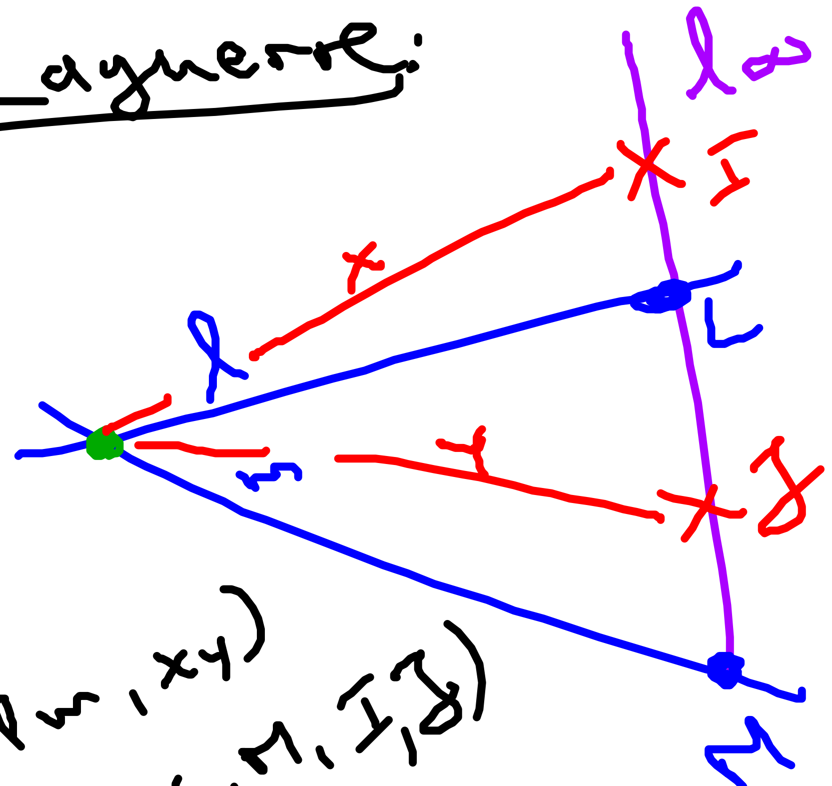
$$|P,Q| = c_{dist} \cdot \ln(P,Q;X,Y)$$





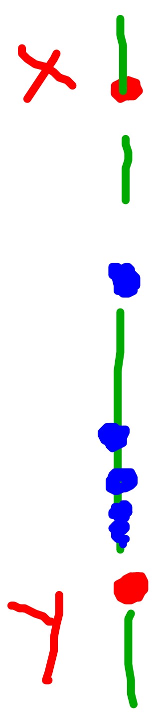
$$\measuredangle(l,m) = c_{ang} \ln(l,m;X,Y)$$

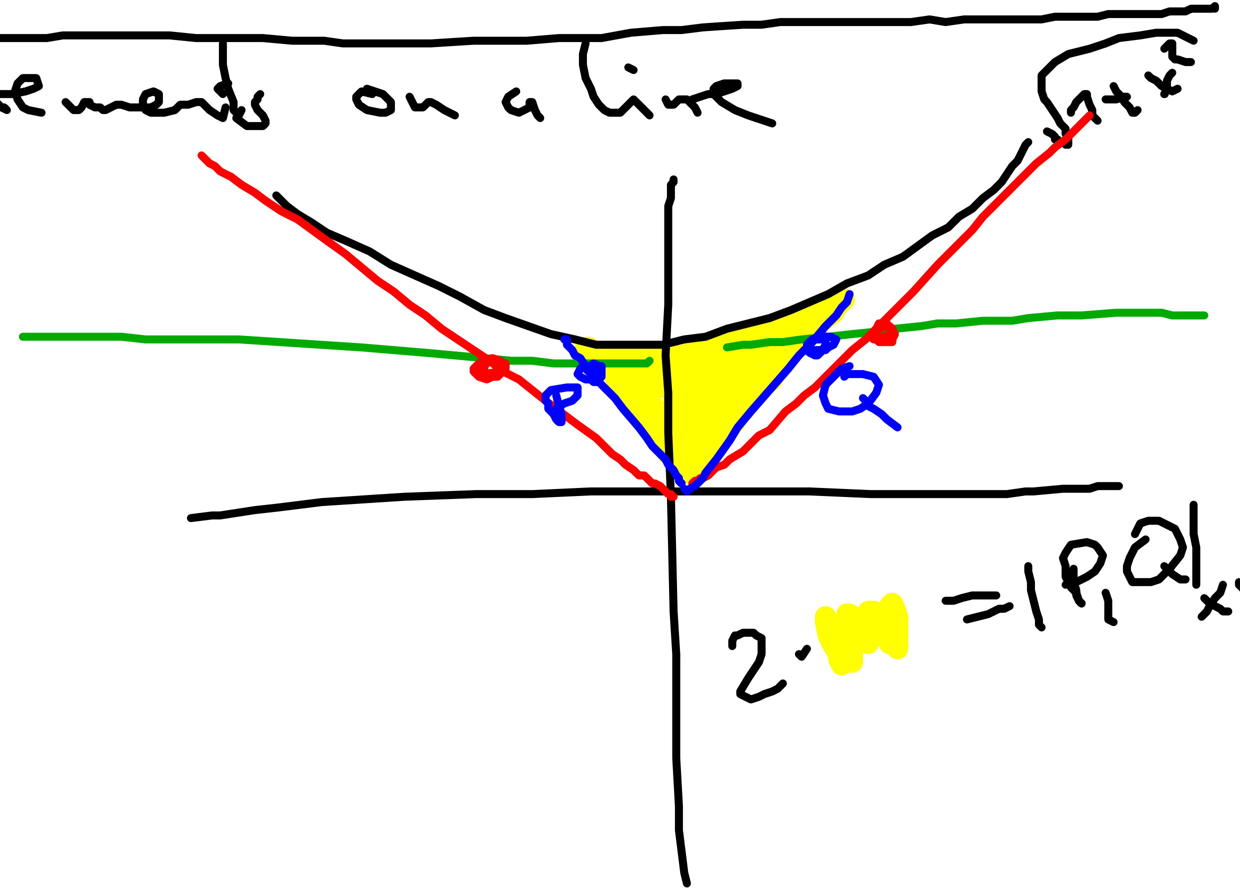
Laguerre:



$(l\,m, XY)$
$= (L,M,I,J)$

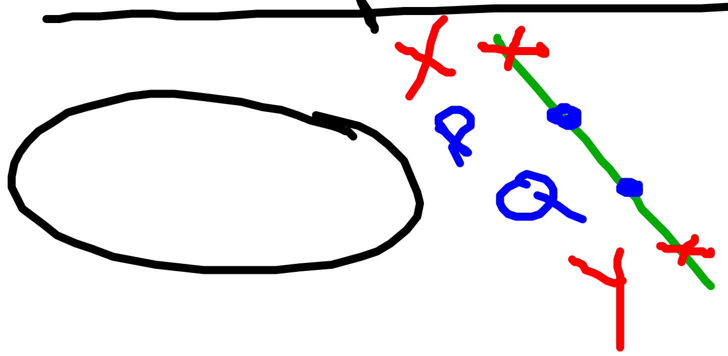
Measurements on a line





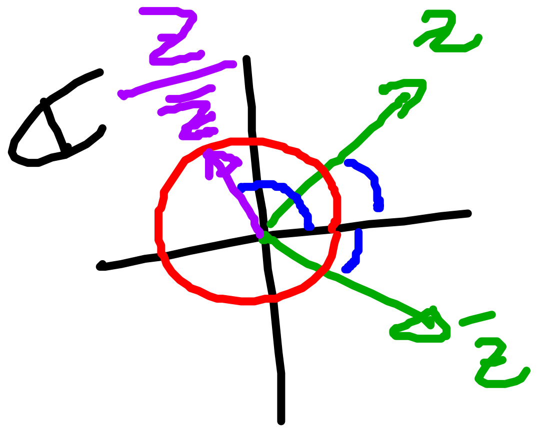
$$2 \cdot \square = |P,Q|_{XY}$$

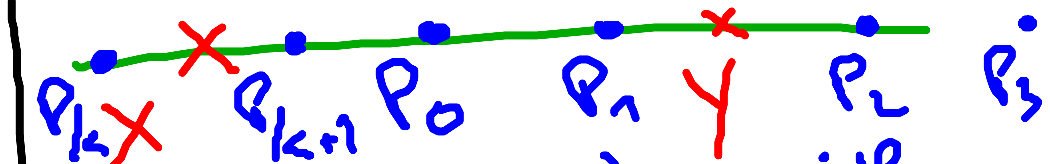
Elliptic measurement: $X = \overline{Y}$



$X = \begin{pmatrix} -i \\ 1 \end{pmatrix}$ $Y = \begin{pmatrix} i \\ 1 \end{pmatrix}$ $P = \begin{pmatrix} p \\ 1 \end{pmatrix}$ $Q = \begin{pmatrix} q \\ 1 \end{pmatrix}$

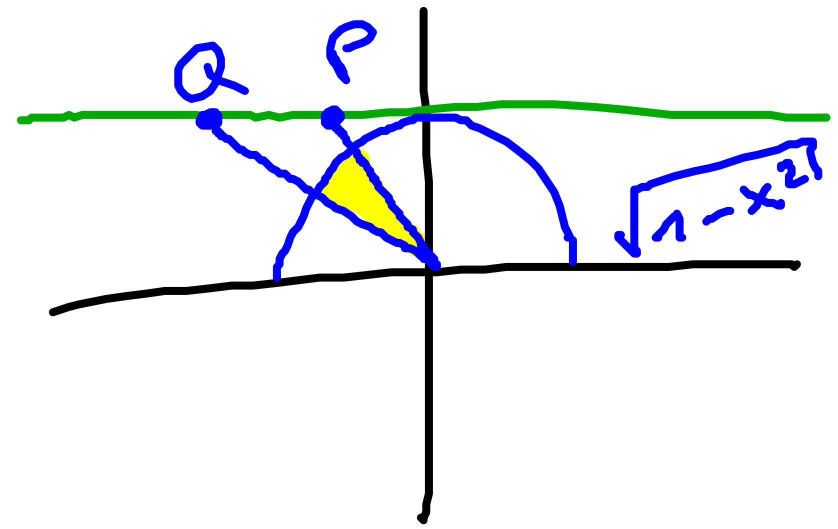
$$|P,Q|_{XY} = c_{dist} \cdot \ln \left(\underbrace{\frac{(p+i)(q-i)}{(p-i)(q+i)}}_{e^{i\varphi}} \right) = c_{dist} \cdot i\varphi$$





$$(P_0, P_1, X, Y) = e^{i\varphi}$$

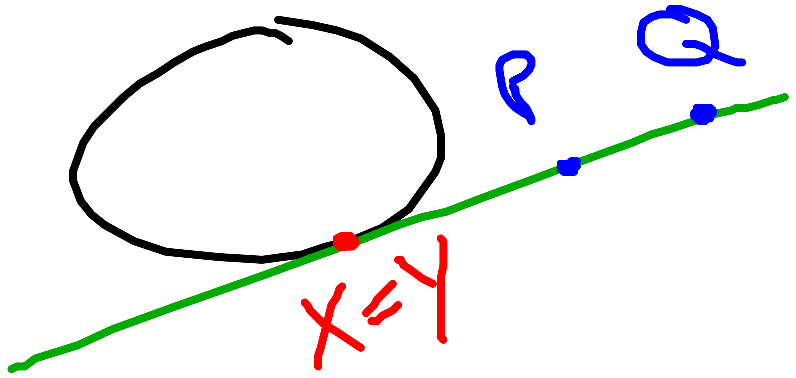
$$(P_0, P_k, X, Y) = (P_0, P_1, X, Y)^k = e^{i\varphi \cdot k}$$



Reasonable choice of c_{dist}:

$$\frac{1}{2i}$$

Parabolic/Euclidean measurement: $X=Y$.



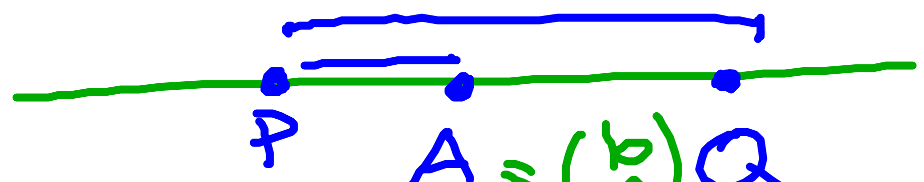
$$|P,Q|_{XX} = c_{dist} \cdot \ln(P,Q,X,X) = 0$$

$\Longrightarrow$ Compare relative distance measures

$X = \begin{pmatrix} 1 \\ -\sqrt{a} \end{pmatrix}$ $Y = \begin{pmatrix} 1 \\ \sqrt{a} \end{pmatrix}$ consider $a \to 0$

$$\ln(\underbrace{P}_{\binom{0}{1}},\underbrace{Q}_{\binom{q}{1}},X,Y) = \ln\left(\frac{q\sqrt{a}-1}{-q\sqrt{a}-1}\right) = \ln(q\sqrt{a}-1) - \ln(-q\sqrt{a}-1)$$

$\Longrightarrow$ Compare lengths



$A = \binom{b}{1}$

$$\lim_{a\to 0} \frac{\ln(P,Q,X,Y)}{\ln(P,A,X,Y)} = \cdots = \frac{q}{b}$$

l'Hospital

Measurements only w.r.t. unit length!

Classification of Cayley-Klein geometries

$\measuredangle(L,M)$ \ $\lvert P,Q\rvert$	Hyp.	Ell.	Euc.
Hyp.	$x^2+y^2-z^2=0$ $x^2+y^2-z^2=0$	$x^2+y^2-z^2=0$ $x^2+y^2-z^2=0$	$z^2=0$ $x^2-y^2=0$ Pseudo Eucl. G. Special Relativity
Ell.	$x^2+y^2-z^2=0$ $x^2+y^2-z^2=0$ Hyperbolic Geom.	$x^2+y^2+z^2=0$ $x^2+y^2+z^2=0$ Elliptic Geom. Geometry on sphere with antipodals ident.	$z^2=0$ $x^2+y^2=0$ I J Eucl. Geom.
Euc.	dual Pseudo Eucl. G.	dual Eucl.	$z^2=0$ $x^2=0$ Galileo Geom.

X, Y are solutions of

$$f = a t_1^2 - t_2^2 \in \mathbb{R}[t_1, t_2]$$

$$\Rightarrow f(X) = 0 = f(Y)$$

a

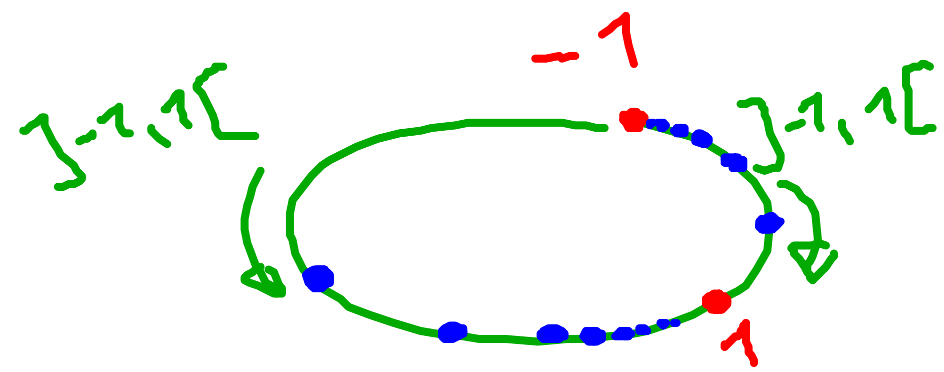
1 Hyperbolic

0 Parabolic

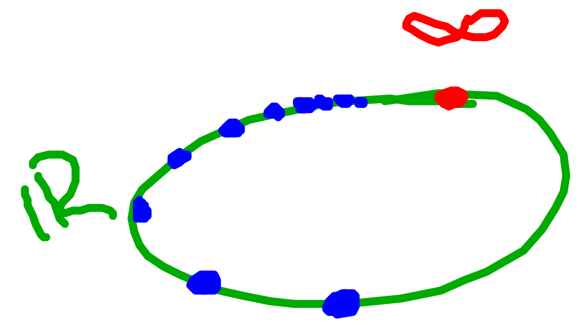
-1 Elliptic

A model

Hyp.:



Euc.:



Ell-