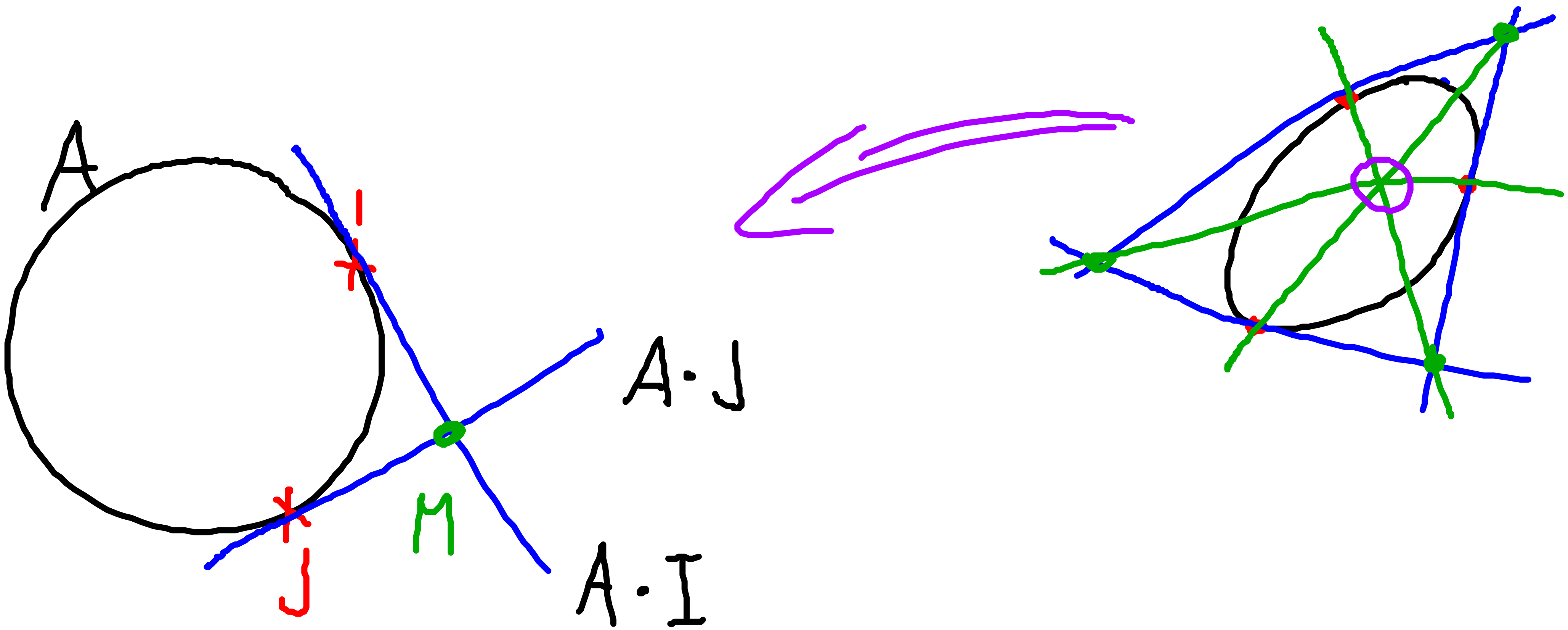
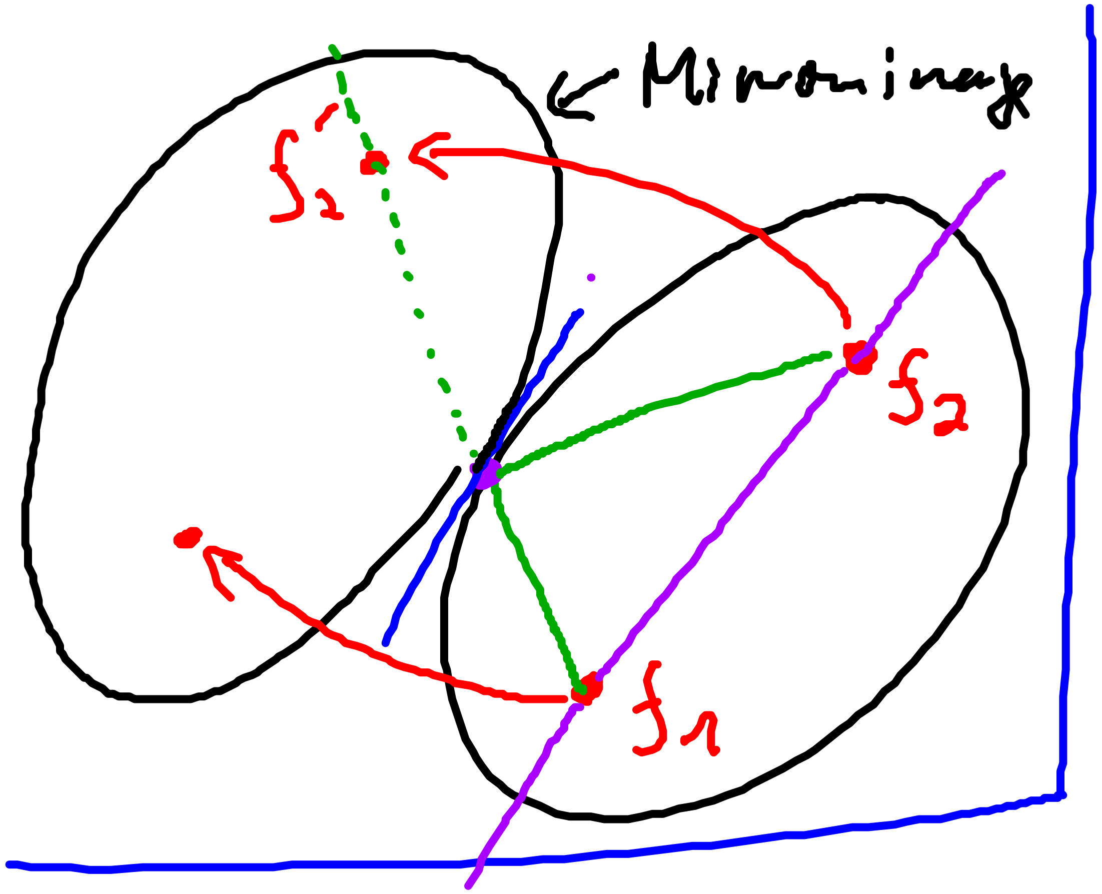
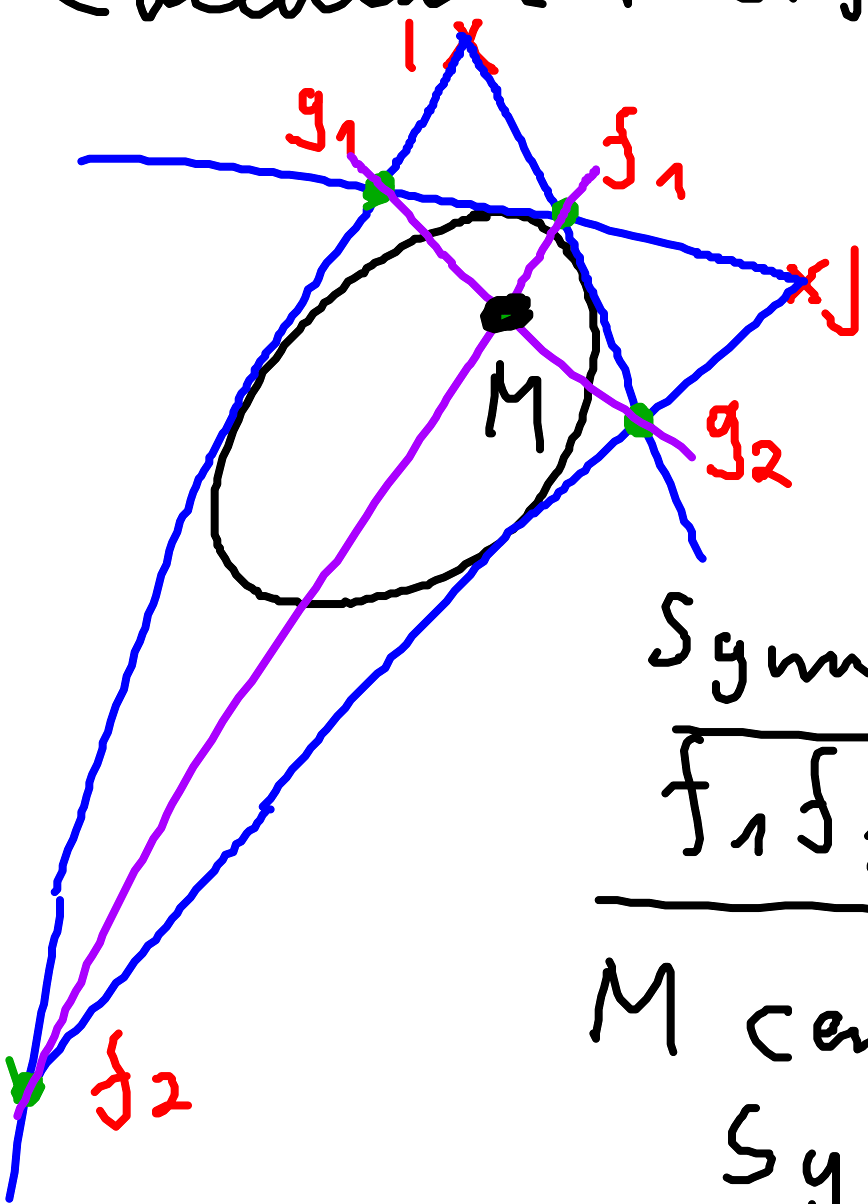
Center of a circle



Foci of a Conic



Calculate Foci projectively

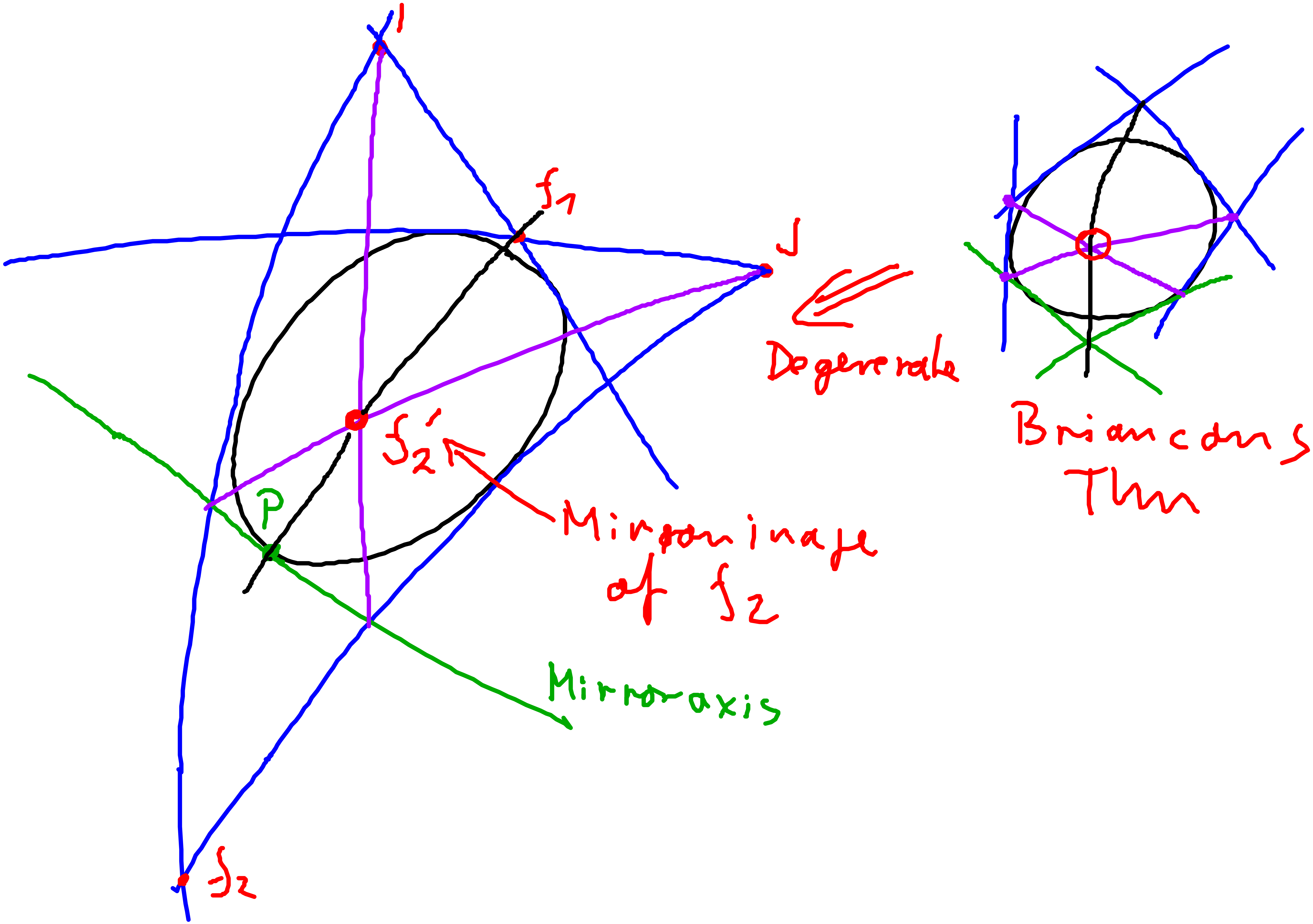


f_1, f_2 a pair of foci

$g_1\ g_2$ a pair of Foci

Symmetry achses $\overline{f_1 f_2}$, $\overline{g_1 g_2}$

M center of Symmetry



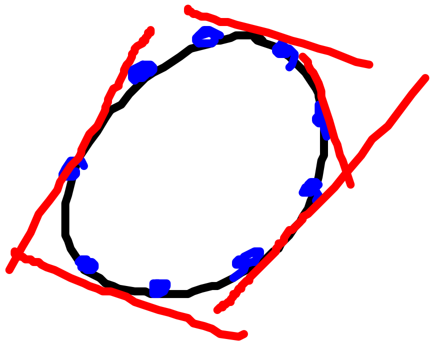
I
f_1
J
f_2'
Mirror image of f_2
P
Mirror axis
f_2
Degenerate
Brianchons Thm

Conics and their Matrices

Conic: $\{p \in \mathcal{P} \mid p^T A p = 0\}$

Dual Conic: $\{\ell \in \mathcal{L} \mid \ell^T B \ell = 0\}$

symmetric

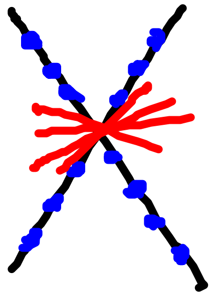


Assume A is invertible:
non-degenerate case

$B = A^{-1} = \lambda \cdot A^{\Delta}, \; \lambda \neq 0$

$A = B^{-1} = \mu B^{\Delta}, \; \mu \neq 0$

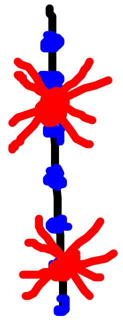
If rank(A) = 2
decomposition into 2 lines



$B = \lambda A^{\Delta}, \; \lambda \neq 0 \leftarrow \operatorname{rank}(B) = 1$

$B^{\Delta} = 0$

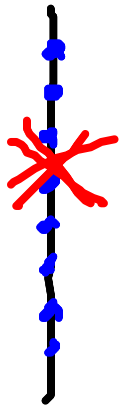
If rank(B) = 2
Double line with 2 special points



$A = \mu B^{\Delta} \quad \mu \neq 0 \leftarrow \operatorname{rank}(A) = 1$

$A^{\Delta} = 0$

If rank(A) = rank(B) = 1
Double line with Double point.



$A^{\Delta} = 0$

$B^{\Delta} = 0$

← Problem
how do A, B relate

Primal Dual Pair: (A, B)

symmetric

A, B are called „primal/dual pair" if $\exists \lambda$ with $\boxed{A \cdot B = \lambda E}$

Case 1: A is invertible: B is multiple of A^{-1}

Case 2: $\operatorname{rank}(A) = 2$:

Exmpl:

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\lambda \cdot E}$$

Case 3: $\operatorname{rank}(B) = 2$: similar

Case 4: $\operatorname{rank}(A) = \operatorname{rank}(B) = 1$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix}$$

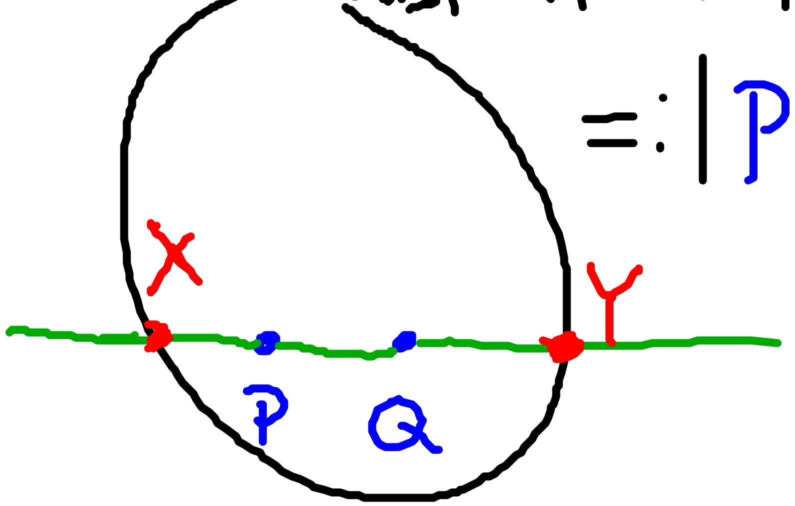
New Topic: Cayley Klein Geometries

- They define measurements of angles, distances
- Are based on a Conic.

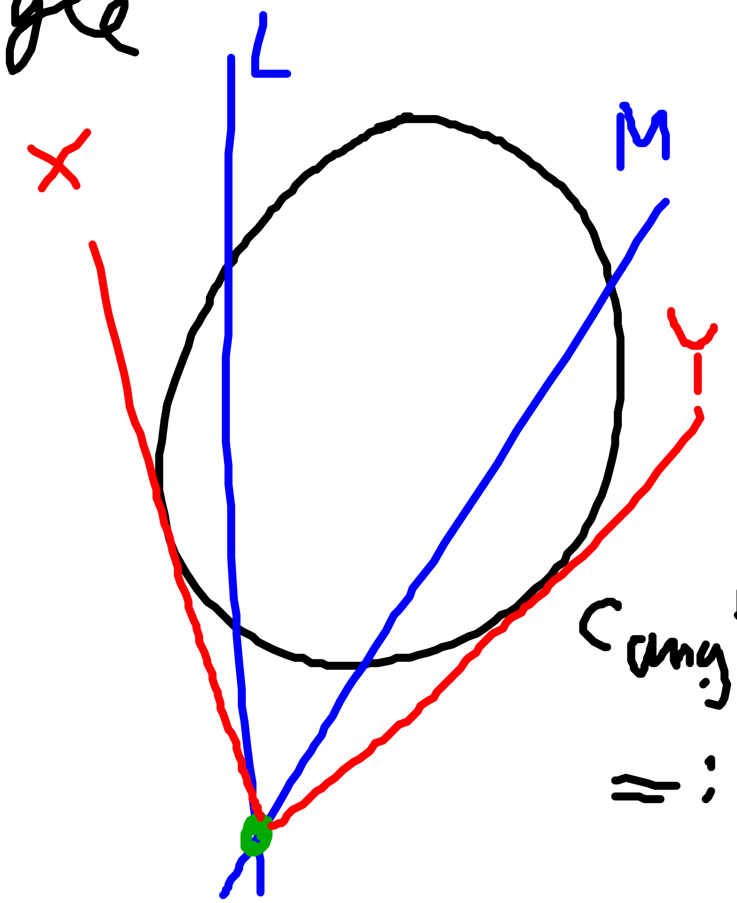
Cooking recepie: Ingredient: • Conic (A, B)
• 2 constants, c_{ang}, c_{dist}

Distance

$$c_{dist} \cdot \ln(P,Q;X,Y) =: |P,Q|$$



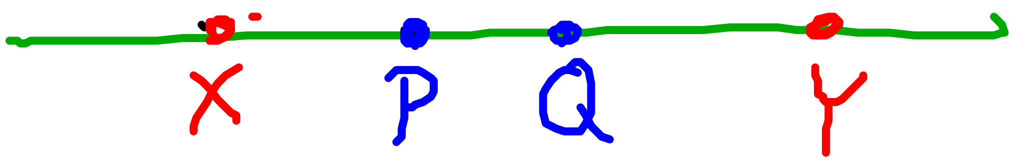
Angle



$$c_{ang} \ln(L,M;X,Y) =: \measuredangle L,M$$

First consider Distances on a fixed Line

$$|P,Q|_{X,Y} = c_{\text{dist}} \ln(PQ, XY)$$



3 different cases: X, Y are real $\Rightarrow$ hyperbolic case

X, Y are complex conjugates $\Rightarrow$ elliptic case

X = Y is real $\Rightarrow$ euclidean case

- $|P,Q|_{XY} = -|Q,P|_{XY}$ since $(PQ,XY) = \dfrac{1}{(QP,XY)}$
- $|P,P|_{X,Y} = 0$ since $(PP,XY) = 1$
- $|PQ|_{XY} + |QR|_{XY} = |PR|_{XY}$ since $(PQ,XY)\cdot(QR,XY) = (PR,XY)$