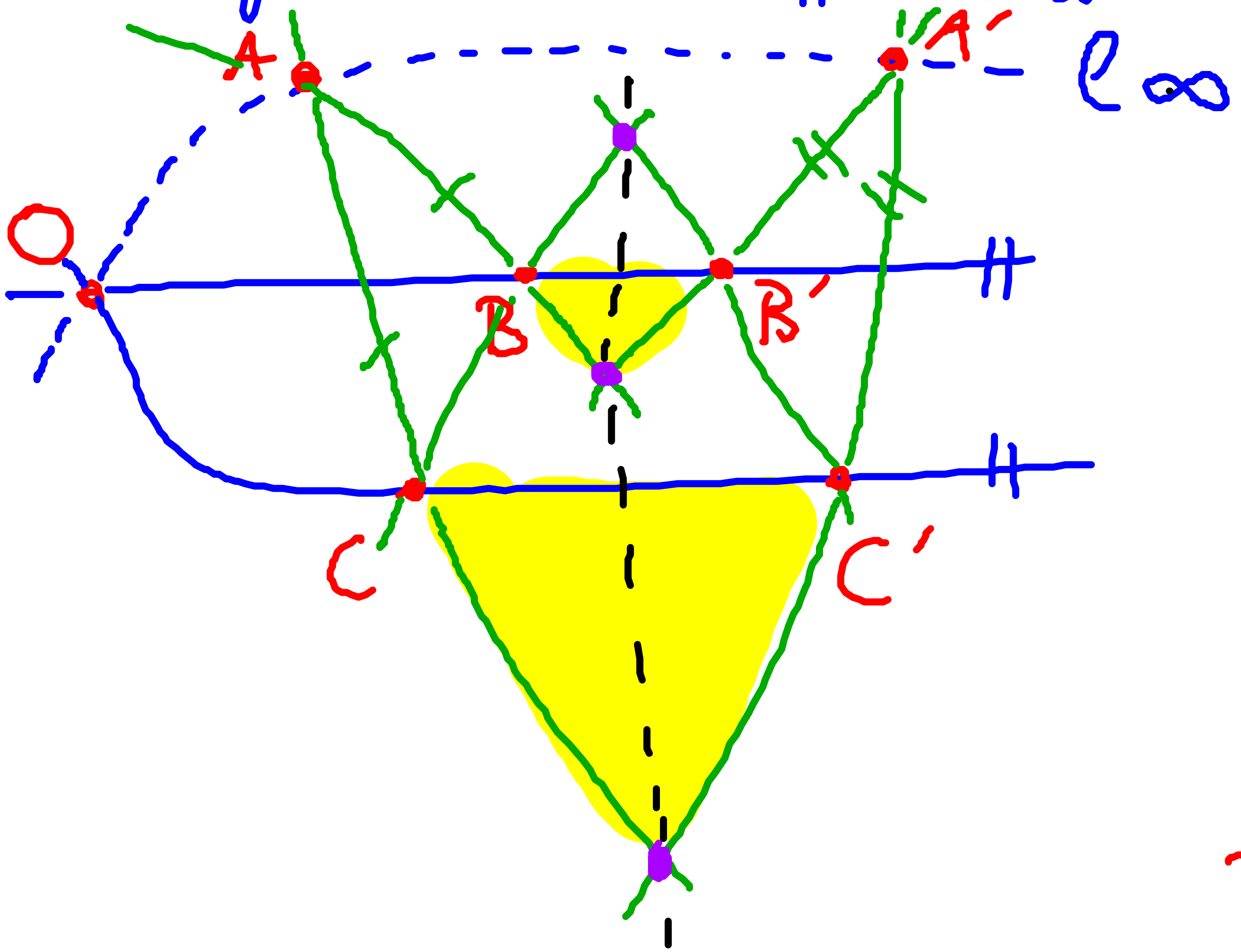
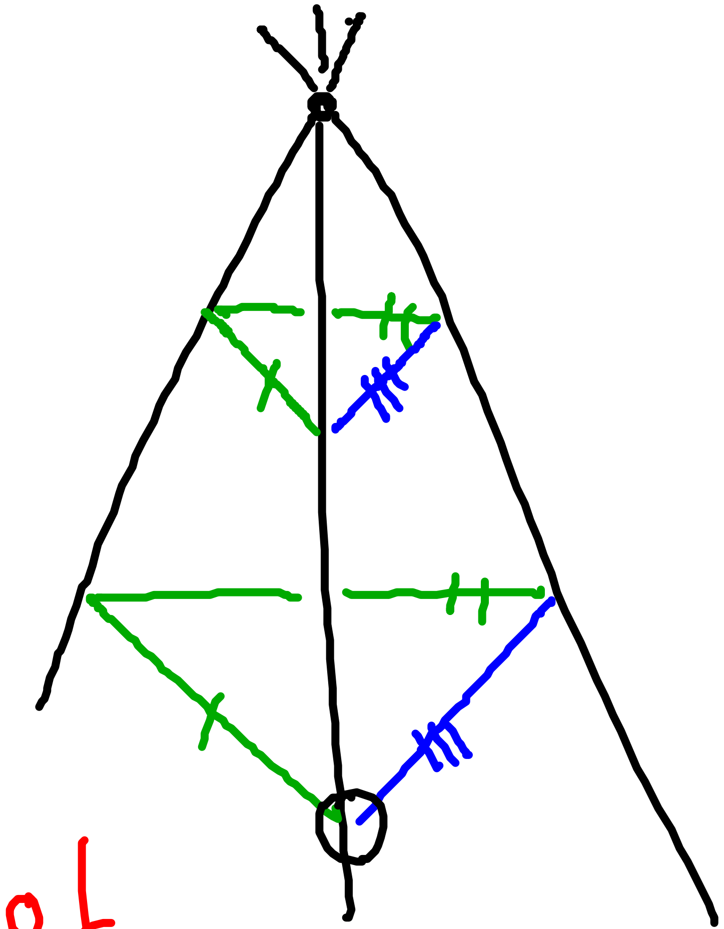
2. Proof strategy: Chose a special situation to get a nice "euclidean" Theorem



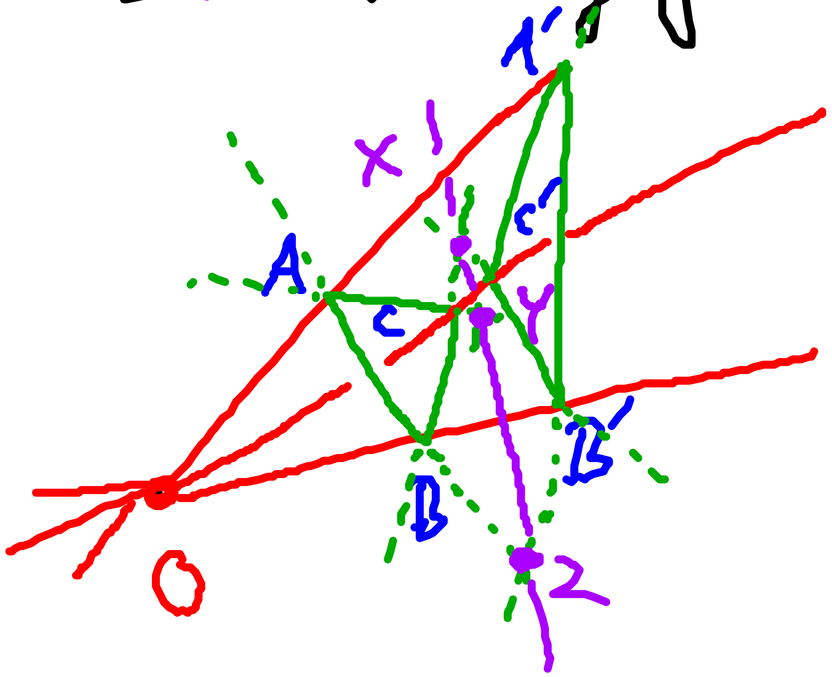


Proof

$\Rightarrow$ Exercises

3. Stategy: Interpret the drawing in 3D-space

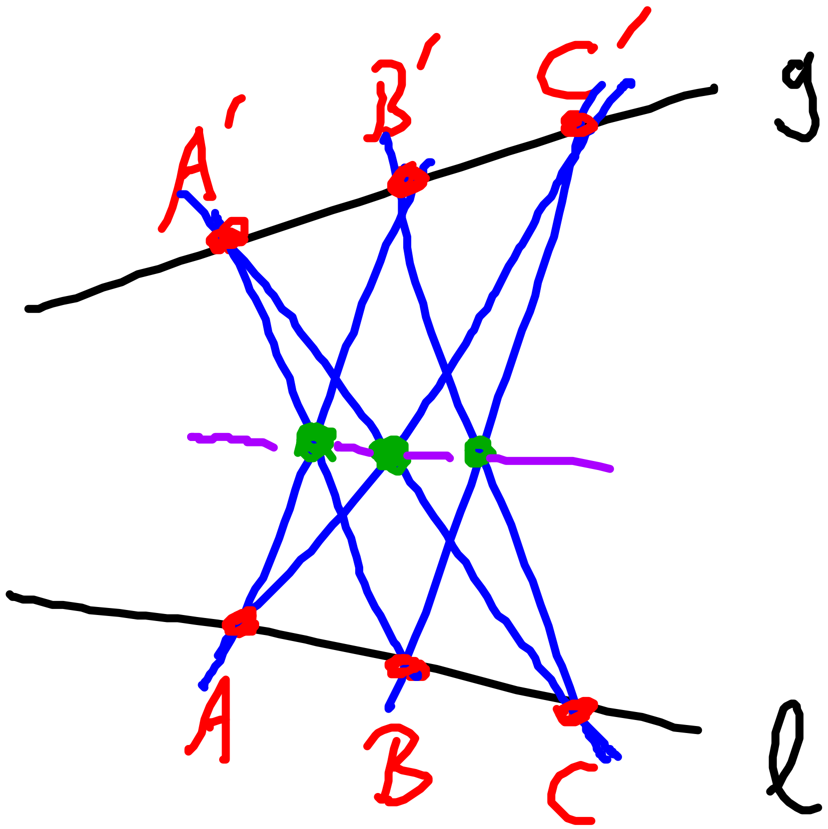
(this works in every ($\mathbb{P}_{\mathbb{K}}, \mathcal{L}_{\mathbb{K}}, \mathcal{I}_{\mathbb{K}}$))



- Consider the three lines trough O as lines in 3-space not all in a common plan
- The triangles A, B, C and A', B' C' are triangels in 3-space
- These triangles „live" in certain Planes
- These two planes intersect in a line
- X, Y, Z are on this line

Pappos's

Let g, ℓ be two distinct lines.



Let A, B, C be three pts on ℓ

Let A', B', C' be three pts on g

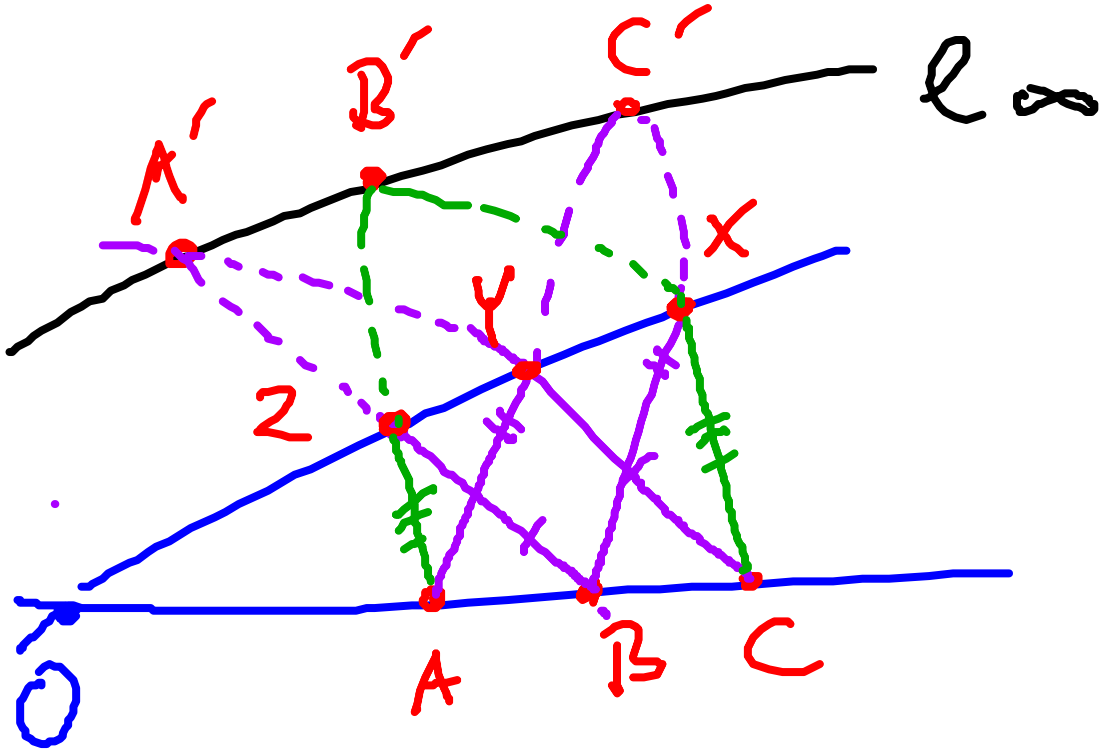
All Point distinct, and distint from $g \wedge \ell$.

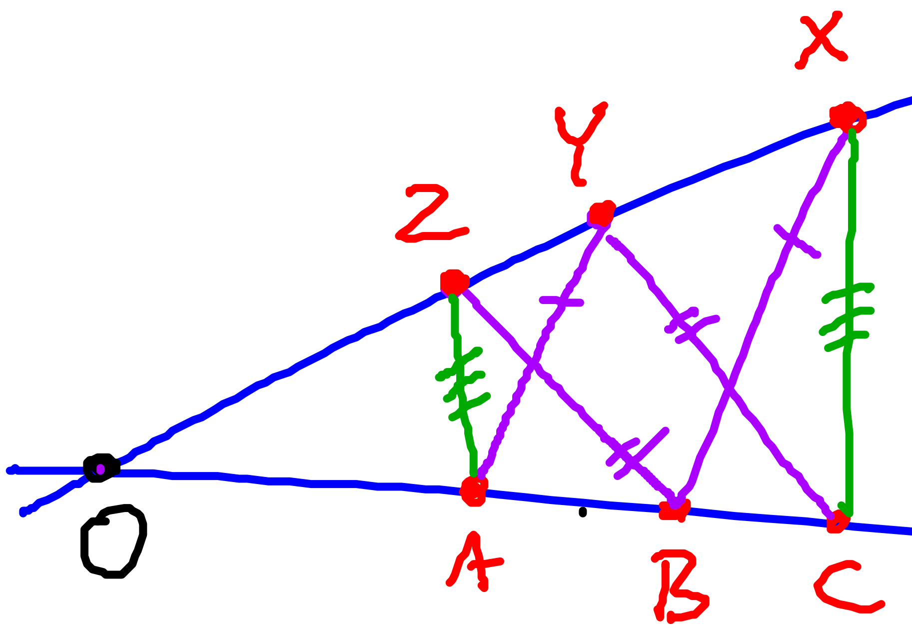
$\Rightarrow$ $(A \vee B') \wedge (A' \vee B)$

$(B \vee C') \wedge (B' \vee C)$

$(C \vee A') \wedge (C' \vee A)$

are collinear





• use oriented lengths.
• Don't divide by zero

Euklidean version of Pappos

- A, B, C collinear
- X, Y, Z collinear
- $AY \parallel BX$, $BZ \parallel CY$

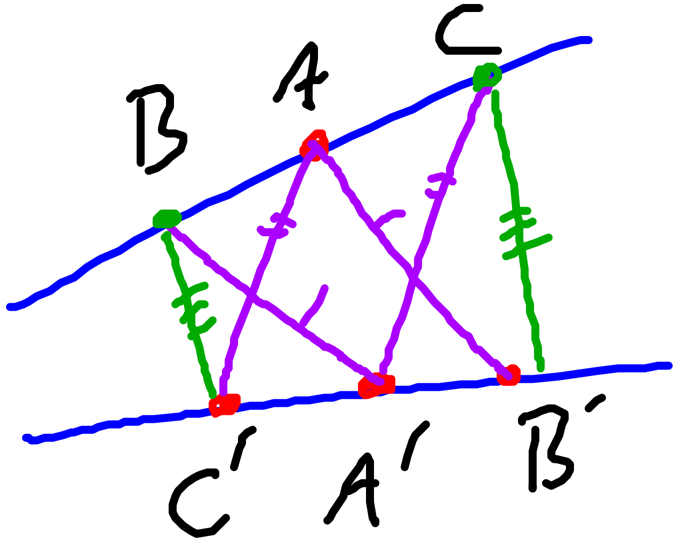
$\Rightarrow AZ \parallel CX$

Proof: $AY \parallel BX \Rightarrow \boxed{\dfrac{OA}{OB} = \dfrac{OY}{OX}}$

$BZ \parallel CY \Rightarrow \boxed{\dfrac{OB}{OC} = \dfrac{OZ}{OY}}$

$\Downarrow$

$AZ \parallel CX \Leftarrow \boxed{\dfrac{OA}{OC} = \dfrac{OZ}{OX}}$



$AB' \parallel BA'$

$AC' \parallel CA'$

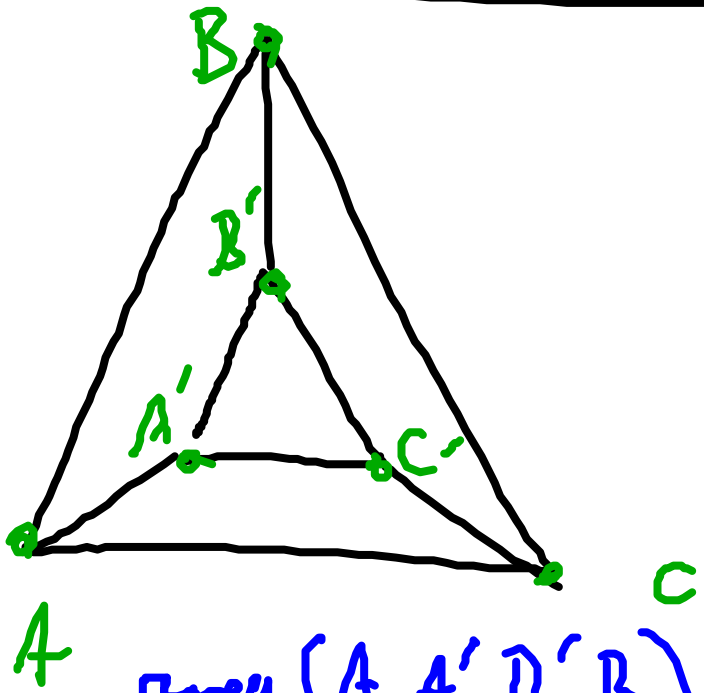
ABC collinear

$A'B'C'$ collinear

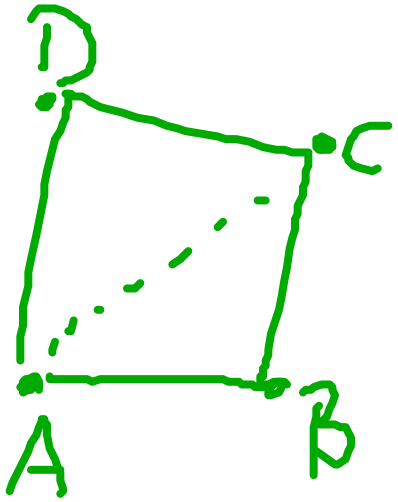
$\Rightarrow BC' \parallel CB'$

$\text{area}(ABC) = 0$

$\Longleftrightarrow A, B, C$ collinear



$\text{area}(AA'B'B)$
$+ \text{area}(CC'A'A)$
$+ \text{area}(BB'C'C)$
$+ \text{area}(A'C'B')$
$+ \text{area}(ABC) = 0$



$\text{area}(ABCD)$
$= \text{area}(ABC)$
$+ \text{area}(CDA)$

$\text{area}(ABCD) = 0$

$\Longleftrightarrow$