

Lineare Algebra und Analytische Geometrie II

Klassifikation der Quadriken in den affinen Räumen $A^n(V^n, \mathbb{R})$ ($n = 1, 2, 3$)

A^1 (Quadriken in einer affinen Geraden)

	r	s	Normalform	Name
(R1)	1	1	$x_1^2 = 0$	Doppelpunkt
(R2)	-	-	-	kein Exemplar, da $r > 0$ nicht erfüllbar
(R3)	1	1	$x^2 + 1 = 0$	NF in $A^1(V^1, \mathbb{R})$ nicht erfüllbar
	1	-1	$-x_1^2 + 1 = 0$	Punktpaar

A^2 (Quadriken in einer affinen Ebene)

(R1)	2	2	$x_1^2 + x_2^2 = 0$	Doppelpunkt
	2	0	$x_1^2 - x_2^2 = 0$	schneidendes Geradenpaar
	1	1	$x_1^2 = 0$	Doppelgerade
(R2)	1	1	$x_1^2 + 2x_2 = 0$	Parabel
(R3)	2	2	$x_1^2 + x_2^2 + 1 = 0$	NF in $A^2(V^2, \mathbb{R})$ nicht erfüllbar
	2	0	$x_1^2 - x_2^2 + 1 = 0$	Hyperbel
	2	-2	$-x_1^2 - x_2^2 + 1 = 0$	Ellipse
	1	1	$x_1^2 + 1 = 0$	NF in $A^2(V^2, \mathbb{R})$ nicht erfüllbar
	1	-1	$-x_2^2 + 1 = 0$	paralleles Geradenpaar

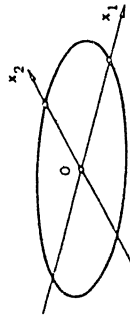
A^3 (Quadriken in einem affinen Raum)

(R1)	3	3	$x_1^2 + x_2^2 + x_3^2 = 0$	Doppelpunkt
	3	1	$x_1^2 + x_2^2 - x_3^2 = 0$	Kegel
	2	2	$x_1^2 + x_2^2 = 0$	Doppelgerade
	2	0	$x_1^2 - x_2^2 = 0$	schneidendes Ebenenpaar
	1	1	$x_1^2 = 0$	Doppelebene
(R2)	2	2	$x_1^2 + x_2^2 + 2x_3 = 0$	elliptisches Paraboloid
	2	0	$x_1^2 - x_2^2 + 2x_3 = 0$	hyperbolisches Paraboloid
	1	1	$x_1^2 + 2x_2 = 0$	parabolischer Zylinder

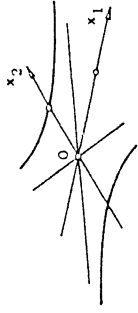
(R3)	3	3	$x_1^2 + x_2^2 + x_3^2 + 1 = 0$	NF in $A^3(V^3, \mathbb{R})$ nicht erfüllbar
	3	1	$x_1^2 + x_2^2 - x_3^2 + 1 = 0$	zweischaliges Hyperboloid
	3	-1	$x_1^2 - x_2^2 - x_3^2 + 1 = 0$	einschaliges Hyperboloid
	3	-3	$-x_1^2 - x_2^2 - x_3^2 + 1 = 0$	Ellipsoid
	2	2	$x_1^2 + x_2^2 + 1 = 0$	NF in $A^3(V^3, \mathbb{R})$ nicht erfüllbar
	2	0	$x_1^2 - x_2^2 + 1 = 0$	hyperbolischer Zylinder
	2	-2	$-x_1^2 - x_2^2 + 1 = 0$	elliptischer Zylinder
	1	1	$x_1^2 + 1 = 0$	NF in $A^3(V^3, \mathbb{R})$ nicht erfüllbar
	1	-1	$-x_1^2 + 1 = 0$	paralleles Ebenenpaar



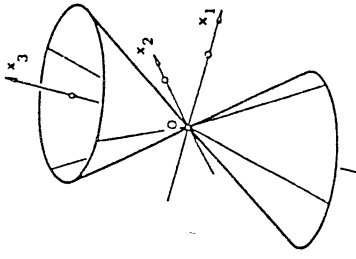
Parabel: $x_1^2 + 2x_2 = 0$



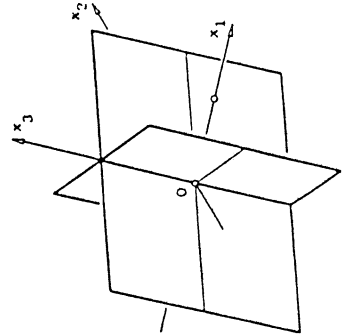
Ellipse: $-x_1^2 - x_2^2 + 1 = 0$



Hyperbel: $x_1^2 - x_2^2 + 1 = 0$

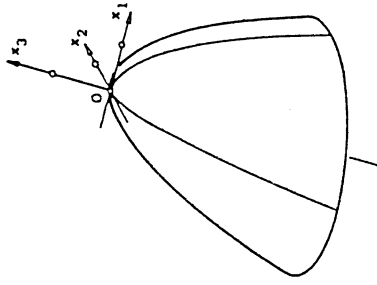


Kegel: $x_1^2 + x_2^2 - x_3^2 = 0$

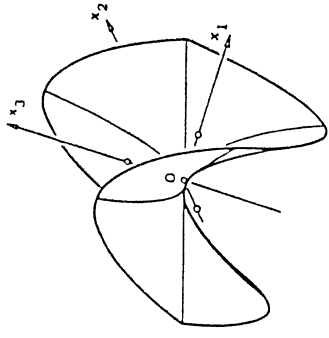


schneidendes Ebenenpaar: $x_1^2 - x_2^2 = 0$

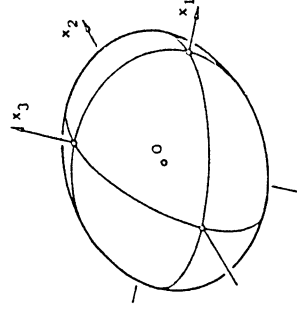
Doppelebene: $x_1^2 = 0$



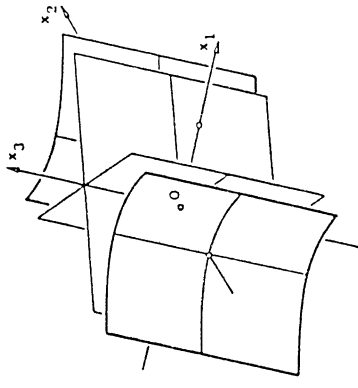
elliptisches Paraboloid: $x_1^2 + x_2^2 + 2x_3 = 0$



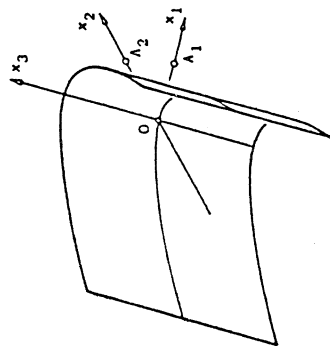
hyperbolisches Paraboloid: $x_1^2 - x_2^2 + 2x_3 = 0$



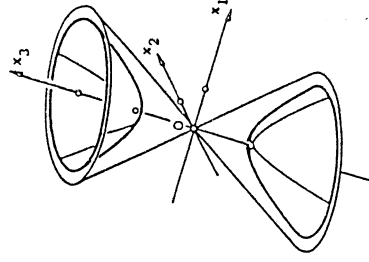
Ellipsoid: $-x_1^2 - x_2^2 - x_3^2 + 1 = 0$



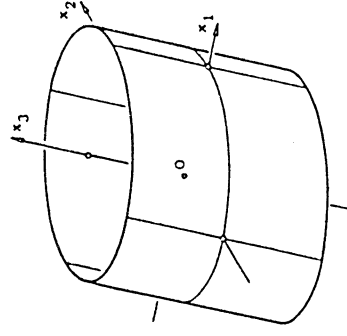
hyperbolischer Zylinder: $x_1^2 - x_2^2 + 1 = 0$



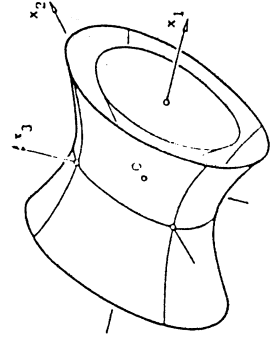
parabolischer Zylinder: $x_1^2 + 2x_2 = 0$



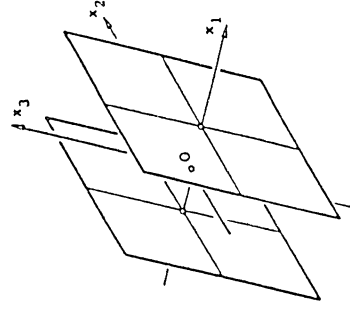
zweischaliges Hyperboloid: $x_1^2 + x_2^2 - x_3^2 + 1 = 0$



elliptischer Zylinder: $-x_1^2 - x_2^2 + 1 = 0$



einschaliges Hyperboloid: $x_1^2 - x_2^2 - x_3^2 + 1 = 0$



paralleles Ebenenpaar: $-x_1^2 + 1 = 0$